

Comment on ‘Generalized Heisenberg algebra coherent states for power-law potentials’

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We argue that the statistical features of generalized coherent states for power-law potentials based on Heisenberg algebra, presented in a recent paper by Berrada *et al* [1] are incorrect.

In a recent article, Berrada *et al* [1] constructed generalized Heisenberg algebra (GHA) coherent states for power-law potentials [2]. Following the earlier analysis of power-law potentials [3, 4], the authors investigated the GHA coherent states for loosely binding potentials and tightly binding potentials. It is claimed that GHA coherent states for loosely binding potentials ($k < 2$) exhibit a super-Poissonian behavior for lower values of $|z|$ followed by sub-Poissonian behavior for higher $|z|$ values. On the other hand, GHA coherent states of tightly binding potentials ($k > 2$), always exhibit the sub-Poissonian distribution. However, in this case the states for lower values of k (i.e., $k = 5$ and $k = 10$) are declared less classical as $|z|$ becomes large which means that the states in question get farther from the states exhibiting Poissonian statistics, in particular Glauber's coherent states [5]. In this comment, we argue that the transition of super-Poissonian to sub-Poissonian behavior in GHA coherent states for loosely binding potentials and less classical behavior in GHA coherent states of tightly binding potentials for $k = 5$ and $k = 10$, presented in ref. [1], are incorrect and are a consequence of wrong numerics.

In order to discuss the results presented by the authors, we rewrite the generalized Heisenberg algebra coherent states for power-law potentials [1]

$$|z, k\rangle = N(|z|, k) \sum_{n=0}^{\infty} \frac{z^n}{\sqrt{g(n, k)}} |n\rangle, \quad (1)$$

where

$$g(n, k) = \prod_{j=1}^n \left[\left(j + \frac{\gamma}{4} \right)^{2k/(k+2)} - \left(\frac{\gamma}{4} \right)^{2k/(k+2)} \right], \quad (2)$$

and the normalization function

$$N(|z|, k) = \left(\sum_{n=0}^{\infty} \frac{|z|^{2n}}{g(n, k)} \right)^{-1/2}. \quad (3)$$

The statistical behavior of GHA coherent states of power-law potentials can be probed through the weighting distribution,

$$P_n(|z|, k) \equiv |\langle n | z, k \rangle|^2 = N^2(|z|, k) \frac{|z|^{2n}}{g(n, k)}. \quad (4)$$

However, in Ref. [1], Mandel's Q -parameter has been used to determine the nature of weighting distributions of these states. This parameter is defined as [6]

$$Q = \frac{\sigma^2}{\langle \hat{N} \rangle} - 1, \quad (5)$$

where, $\langle \hat{N} \rangle$ is the mean and σ^2 is variance, i.e.,

$$\sigma^2 = \langle \hat{N}^2 \rangle - \langle \hat{N} \rangle^2$$

of the corresponding distribution. The Mandel's Q -parameter is defined such that, the weighting distribution of coherent states is Poissonian if $Q = 0$, super-Poissonian if $Q > 0$ and sub-Poissonian if $Q < 0$. However, it is obvious from Eq. (5) that in order to calculate Mandel's Q -parameter one needs to calculate the expectation values,

$$\langle z, k | \hat{N}^m | z, k \rangle = N^2(|z|, k) \sum_{n=0}^{\infty} n^m \frac{|z|^{2n}}{g(n, k)} \equiv \langle \hat{N}^m \rangle, \quad (6)$$

for $m = 1, 2$, where \hat{N} is bosonic number operator. Therefore, Eq. (6) serves as synthesis equation for mean, variance and Mandel's parameter.

In Figs. 1 and 2 of reference [1], Mandel's Q-parameter for GHA coherent states of loosely binding potentials ($k < 2$) and tightly binding potentials ($k > 2$), respectively, is plotted versus coherent state amplitude $|z|$. The plots in Fig. 1 show that Mandel's Q-parameter undergoes transition from positive values to negative ones after a short range of $|z|$, afterwards it attains a steady state value that approaches to -1 . As a result, the weighting distribution is reported to undergo a transition from super-Poissonian to sub-Poissonian. On the other hand, the plots in Fig. 2 show that Mandel's Q-parameter is negative for all values of $|z|$ (sub-Poissonian distribution), however, for lower values of k (i.e., $k = 5$ and $k = 10$) it sharply tends to reach its minimum value $Q = -1$ at large values of $|z|$. The transition of Mandel's parameter from positive values to negative ones in Fig. 1 and a sharp deviation from nearly steady state towards its minimum value $Q = -1$ (in case of $k = 5$ and $k = 10$) in Fig. 2 is very unexpected. In the following, we present our analytical as well as numerical arguments against these results.

As a preliminary bit of information, we argue that Mandel's Q -parameter depends on variance-to-mean ratio (VMR) such that $Q = -1$ when $VMR = 0$ as expressed by Eq. (5). In order for $VMR = 0$ it is required that either variance of the distribution should be zero ($\sigma^2 = 0$) or mean of the distribution should take an infinitely large value ($\langle \hat{N} \rangle \rightarrow \infty$) in comparison to the variance. It is obvious from the synthesis equation (Eq. (6)) that

expectation values of \hat{N} are directly proportional to $|z|$. Consequently, for a finite value of $|z|$, the mean of the distribution takes finite values and variance takes non-zero positive value. Therefore, neither of the situations that lead to $Q = -1$ is possible in the case of GHA coherent states for power-law potentials. In order to validate our analytical argument, we use the synthesis equation (Eq. (7)) to compute the values of mean, variance and Mandel's parameter of GHA coherent states of a loosely binding potential defined by $k = 1.5$ for various values of $|z|$ which are given in Table 1. We find that, Mandel's Q -parameter takes positive values i.e., 0.162 and 0.164 for $|z| = 12.5$ and $|z| = 12.5$, respectively, in contrast to the results displayed in Fig. 1 of Ref. [1] for $k = 1.5$. Using the same equation, we can check the values of Mandel's parameter for the plots of other values of k mentioned above ($k = 0.5, 1$ in Fig. 1 and $k = 5, 10$ in Fig. 2). Our analytical and numerical arguments are in complete agreement with earlier published results [2]. These facts lead us to conclude that the results displayed by plots in Fig. 1 and by plots for $k = 5, 10$ in Fig. 2 of [1] are incorrect.

As a matter of fact, we probe the possible error in the numerical computation of Mandel's Q -parameter that led to the wrong conclusions in Ref.[1]. It is seen that synthesis equation (Eq. (6)) for mean, variance and Mandel's parameter consists of the infinite sum of terms. In numerical computation of mean, variance and Mandel's parameter, one has to replace the infinite sum involved in Eq. (6) by a finite sum. This finite sum is obtained by truncating the summation at a suitable $n = n_{max}$, such that, $n_{max} \geq n_{th}$ where n_{th} is

$ z $	$\langle \hat{N} \rangle$	σ^2	Q	n_{max}
2.5	9.44 (9.44)	10.05 (10.05)	0.064 (0.064)	50
5.0	43.94 (43.94)	50.06 (50.06)	0.139 (0.139)	100
7.5	111.45 (111.43)	128.67 (127.79)	0.154 (0.147)	200
10.0	216.92 (147.57)	251.58 (7.63)	0.160 (-0.948)	400
12.5	364.20 (149.14)	423.31 (1.56)	0.162 (-0.989)	600
15.0	556.57 (149.52)	647.64 (0.69)	0.164 (-0.995)	700

Table 1: The values without brackets are calculated by taking various values of n_{max} , such that, the condition $n_{max} > n_{th}$ is satisfied, whereas, the values within brackets are calculated for fixed $n_{max} = 150$.

the threshold beyond which the terms of summation are not contributing significantly. For a particular set of coherent state parameters, the threshold value, n_{th} , for the summation can be probed by hit and trial, such that, the condition $n_{max} \geq n_{th}$ is found to be satisfied if further increase in the cut off value, n_{max} , do not change computed result. However, it is worth noting that each term of the synthesis equation depends on the ratio $|z|^{2n}/g(n, k)$, therefore, the n_{th} (number of significant terms) of the summation increases as $|z|$ increases. As a result, the value of n_{max} should be taken larger while computing Q for greater values of $|z|$ as expressed by Table 1. The situation $n_{max} < n_{th}$, where summation in the synthesis equation is truncated at a value of n beyond which the terms are still contributing significantly leads to the wrong results. The effect of this early truncation is indicated by Table 1 and Fig. 1. It is obvious from the Table 1 and the Fig. 1 that for a particular value of $|z|$, mean, variance and Mandel's parameter take lower values when $n_{max} < n_{th}$ as compared to their corresponding values when all effective terms

are included in the situation i.e, $n_{max} \geq n_{th}$. Moreover, it has been pointed out [2] that the dependence of summation threshold (n_{th}) on $|z|$ increases sharply as the value of power-law exponent decreases. Consequently, the computation of Mandel's parameter for coherent states of potentials defined by lower values of power-law exponent needs high value of n_{max} to include all effective terms of the summation. As an example, we again refer the case of $k = 1.5$ given in Table 1, where, $n_{th} > 650$ for $|z| = 15$ and we have taken $n_{max} = 700$ to compute the Q , correctly. In order to calculate Q for high values of $|z|$ (e.g, up to the range in ref. [1]) a very high value of n_{max} would be needed that may exceed the computational limit of ordinary computers. For the reason, the results presented in Fig. 1 and Fig. 2 ($k = 5, 10$) for higher values of $|z|$ are questionable as far as their correct computation is concerned. However, from Table 1 and Fig. 1 it can be inferred easily that if it were possible to perform computation with very high value of n_{max} so that $n_{max} \geq n_{th}$, then Q would go on increasing without abrupt deviation towards -1 as $|z|$ increases. These facts leads us to conclude that the abrupt deviation of Mandel's parameter towards negative values displayed by Fig. 1, and the abrupt deviation of Mandel's parameter in plots for $k = 5$ and $k = 10$ towards -1 displayed by Fig. 2 is nothing but truncation error, such that, authors truncated the infinite sum at $n_{max} < n_{th}$.

As a conclusion, it is stated that GHA coherent states for loosely binding potentials ($k < 2$) exhibit a super-Poissonian distribution for all values of $|z|$ without any transition to sub-Poissonian behavior at higher values of $|z|$

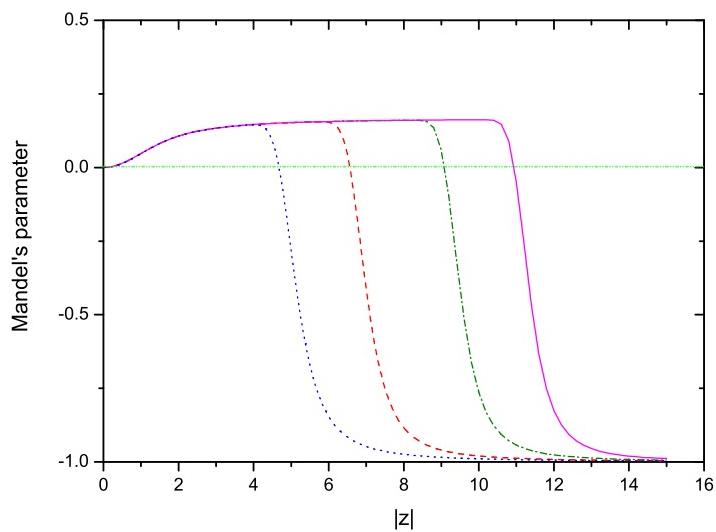


Figure 1: Mandel's Q-parameter for $k = 1.5$ versus $|z|$ for $n_{max} = 50$ (dotted line), $n_{max} = 100$ (dashed line), $n_{max} = 200$ (dashed-dotted line), $n_{max} = 300$ (solid line).

and GHA coherent states for tightly binding potentials ($k > 2$), exhibit a sub-Poissonian distribution smoothly for all values of $|z|$, without showing less classicality (for $k = 5$ and $k = 10$) at higher values of $|z|$. Our arguments presented in this comment are in complete agreement with the earlier published results [2].

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